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CONCRETE
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Volume LIV, No. 6. LONDON, JUNE, 1959.

EDITORIAL NOTES

"Traffic Engineers": What's in a Name?

FOR as long as most of us remember, or care to remember, the highways of Great Britain have been the responsibility of the Ministry of Transport and the local authorities, whose control has been so great that even a stopping-place for omnibuses cannot be moved without their approval. These bodies decide the routes of new roads and the improvement of existing roads for the purpose of providing a system of highways and other roads that will enable traffic to flow easily. They are also, together with the police, responsible for permitting the storage of cars on the streets of our cities to an extent that seriously hinders the flow of traffic and threatens to stop it completely.

The Minister of Transport seems to think these authorities have been doing their work very badly: he has announced in the House of Commons that it is now considered necessary to have a new bureaucracy called "traffic engineers" after the United States pattern, and to send British "experts" on post-graduate courses to the United States to be instructed in better ways of dealing with the problem. Seldom can a body of Civil servants and employees of local authorities have had such a devastating vote of "no confidence" passed on them as was done by the Parliamentary Secretary of the Ministry of Transport when he stated in the House of Commons last month that "traffic engineering" was one of the American successes, that the Americans had achieved smoother, faster, and safer movement of traffic, and that he wanted to see that done in Britain. It is true that we have no men in this country called traffic engineers, for the good reason that we have hitherto been sensible enough not to use such a misleading description for the people who plan the routes of our new highways and who deal with road and traffic problems generally. Such administrative and planning work is not engineering, and these planners are no more engineers than are engine drivers and mechanics. The men who design the roads and bridges are the engineers, and the planners and the engineers are often employed by the same authority, as in the case of the county councils. In other cases the plans are made by the Ministry or by a local authority and a consulting engineer is called in to design the work. To describe these planners and administrators as engineers conforms to our tendency to depend upon other countries for names as well as ideas.

This pronouncement of the Ministry seems to imply the failure of those responsible for our highways and traffic control to produce new ideas or even

to make proper use of experience gained abroad. The history of the past thirty years indicates that there may be some truth in this suggestion, for it is difficult to think of any improvement in the design and construction of highways and the control of traffic that originated in this country. In the road-building boom of the late 1920's and the 1930's the reinforced concrete road, dual-carriageways, separate cycle-tracks, roundabouts, and " Belisha beacons " were all copied from the U.S.A. or the Continent. Since the war improved road-making machines have been imported and similar machines have been made in this country. We have copied American and Continental types of motor-ways, overhead roads, bridges, and slip-ways to give access to cross roads. Our bureaucracy even uses the American terms " fly-over " to describe a bridge carrying one road over another, " fly-under " for a road passing in a cutting under another road, and, as was recently mentioned in this journal, the U.S. term pavement is being used by Government departments to describe a carriageway. Two white lines along the centre of a road and parking meters were also in use abroad before they were introduced to this country.

The Ministry appears to agree with one of the critics in the House of Commons who complained that the Government had not tackled the traffic problem with the urgency it deserved and that there had been far too little thinking and research, for the Parliamentary Secretary said that the Government " had got to ' sell ' [another Americanism !] to the city authorities the idea of having traffic engineers ". He did not say whether those now responsible for dealing with roads and traffic problems were to be superseded, or whether the planners already engaged on this work were expected to get better results if they were called traffic engineers. It appears, however, that some of the men at present responsible for dealing with our traffic problems are to be sent to the U.S.A. to learn the methods that the Ministry wishes to see in use in Great Britain. The Minister of Transport has already decided that some of his officers are to take a post-graduate course of " traffic engineering " at Yale University, and the London County Council and the Birmingham City Council have also agreed to send members of their staffs to U.S. universities to study the subject. Sir Herbert Manzoni, City Engineer of Birmingham, is reported as saying that " The U.S.A. has brought the science of traffic engineering to a further stage of tabulation than we have, and I think we ought to have the advantage of their systematising methods "—a jumble of words as chaotic as the jumble of traffic in our cities and on our main highways at week-ends.

It seems to us that the traffic problems here will not be solved merely by sending men across the Atlantic to find out how they are dealt with in the U.S.A. That knowledge has always been available. Nor will it be solved by describing as traffic engineers those appointed to deal with the problems, or by awarding university degrees in traffic engineering. The solution lies in the provision of more money for the construction of new roads and the improvement of existing roads: in the provision of parking spaces off the roads and the enforcement of their use: in the improvement of the public transport which is now so bad that the use of a car to get to one's place of work is often the only alternative to travelling in a railway compartment in which ten people may be standing in addition to twelve sitting: and in the ruthless cutting out of consultations between a multiplicity of authorities before anything can be done at all.

Design of Helical Staircases.—I.

Statically-Indeterminate Cases.

By JACQUES S. COHEN.

IN a previous article * the values of the tangential, normal, and bi-normal components $\bar{t}$, $\bar{n}$, and $\bar{b}$ of the internal forces and moments at any point were obtained by solving the general differential equation of equilibrium for a twisted curved beam loaded in any direction (*Figs.* 1, 2 and 3) in terms of the radius of curvature ρ, the radius of torsion τ, and the length s of the curve [equations (10), (11) and (12)]. These equations applied to a helix [equation (13) and *Figs.* 5, 6 and 7] gave equations (24) which express the values of T_t, T_n, T_b, M_t, M_n, and M_b at any point of the helix in terms of six constants, $C_1 \ldots C_6$. If the values of the internal forces and moments at one section are known, for example at the support, the constants may be evaluated from equations (24).

The method of solution depends on the conditions at the ends of the beam. For statically-determinate beams (that is cantilevers, simply-supported beams, and in some cases beams with three supports) the forces and moments at the supports may be found from the equations of equilibrium. For statically-indeterminate beams the equations of equilibrium are not sufficient, and equations of deformation and angular rotation at any point are required. In the previous article the case of simply-supported beams was considered ; beams fixed at both ends or fixed at one end and pin-jointed at the other are now considered. The general differential equations of deformation and angular rotation for a twisted curved beam loaded in any direction are established and then applied to the helical curve of a helical staircase (*Figs.* 5, 6 and 7).

The derivations which follow are unavoidably complex, but it should be noted that the results can be applied without a knowledge of the mathematics involved. A study of the examples to be given in a future article will show the simplicity and ease with which any particular staircase may be designed.

General Differential Equations of Deformation and Angular Rotation.

Consider the element ds of a twisted curved beam in equilibrium under the action of external loads and reactions, which cause internal forces and moments T_t, T_n, T_b, M_t, M_n, and M_b, displacements D_t, D_n, and D_b of the centroid of the section, and angular rotations ψ_t, ψ_n, ψ_b, as shown in *Fig.* 3. The deformation of the elementary arc ds, supposing the laws of Hooke and Bernoulli to apply, and ignoring the effects of changes of temperature, are given by (33).

$$\left.\begin{aligned} dD_t &= \frac{T_t}{EA}ds. & \qquad d\psi_t &= \frac{M_t}{GJ}ds. \\ dD_n &= \frac{T_n}{GA'_n}ds. & \qquad d\psi_n &= \frac{M_n}{EI_n}ds. \\ dD_b &= \frac{T_b}{GA'_b}ds. & \qquad d\psi_b &= \frac{M_b}{EI_b}ds. \end{aligned}\right\} \quad . \quad . \quad . \quad (33)$$

* See this journal for May, 1955 in which equations (1) to (32) and *Figs.* 1 to 9 are given.

In these formulæ E is the modulus of elasticity, G the modulus of rigidity (or shear modulus), A the cross-sectional area of the beam, A'_n and A'_b are the areas resisting shear in the principal directions $\bar{n}$ and $\bar{b}$, I_n and I_b are the principal moments of inertia, and J the equivalent polar moment of inertia of the section. J is a function of the geometrical polar moment of inertia I_p, and is given by Saint-Venant as $J = \dfrac{A^4}{40I_p}$. Values of J for various sections, and particularly for rectangular sections, are given in many publications.*

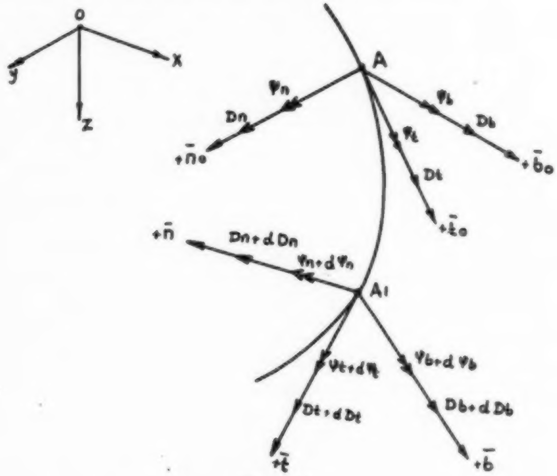


Fig. 10.

Consider two points A and A′ (*Fig.* 10) on the centre-line of a twisted curved beam. If the angular rotation at A is given by the components ψ_t, ψ_n, ψ_b, the corresponding angles at A′ are given by equations (6), and are $\psi_t + \dfrac{ds}{\rho}\psi_n$, $\psi_n - \dfrac{ds}{\rho}\psi_t + \dfrac{ds}{\tau}\psi_b$, and $\psi_b - \dfrac{ds}{\tau}\psi_n$. The change of rotation between A and A′ is therefore given by $+\dfrac{ds}{\rho}\psi_n$, $-\dfrac{ds}{\rho}\psi_t + \dfrac{ds}{\tau}\psi_b$, and $-\dfrac{ds}{\tau}\psi_n$. To these must be added the change of rotation due to the external loading given by the three last equations of (33), the total change of rotation from A to A′ being therefore as in (34).

$$\left.\begin{aligned} d\psi_t &= \frac{ds}{\rho}\psi_n + \frac{M_t}{GJ}ds. \\ d\psi_n &= -\frac{ds}{\rho}\psi_t + \frac{ds}{\tau}\psi_b + \frac{M_n}{EI_n}ds. \\ d\psi_b &= -\frac{ds}{\tau}\psi_n + \frac{M_b}{EI_b}ds. \end{aligned}\right\} \quad . \quad . \quad . \quad (34)$$

* Hütte—Manuel de l'Ingenieur (French edn., 1953), Vol. I, p. 819; Vol. III, p. 324: Dean Peabody—The Design of Reinforced Concrete Structures: H. J. Cowan—Test of the Torsional Strength and Deformation of Rectangular Reinforced Concrete Beams, this journal for February, 1951, p. 51.

Writing $K_1 = \frac{1}{EI_n}$, $K_2 = \frac{1}{EI_b}$, and $\sigma = \frac{I_n}{J}\frac{E}{G}$, equation (34) becomes (35).

$$\left.\begin{aligned} d\psi_t &= \frac{ds}{\rho}\psi_n + K_1\sigma M_t\,ds. \\ d\psi_n &= -\frac{ds}{\rho}\psi_t + \frac{ds}{\tau}\psi_b + K_1 M_n\,ds. \\ d\psi_b &= -\frac{ds}{\tau}\psi_n + K_2 M_b\,ds. \end{aligned}\right\} \quad . \quad . \quad . \quad (35)$$

A displacement D_t, D_n, D_b at A will give a displacement at A′ which is given from equation (6) by $D_t + D_n\frac{ds}{\rho}$, $D_n - D_t\frac{ds}{\rho} + D_b\frac{ds}{\tau}$, $D_b - D_n\frac{ds}{\tau}$. Therefore the change of displacement is given by (35a).

$$D_n\frac{ds}{\rho},\ -D_t\frac{ds}{\rho} + D_b\frac{ds}{\tau},\ -D_n\frac{ds}{\tau}. \quad . \quad . \quad . \quad (35a)$$

The angular rotation (ψ_t, ψ_n, ψ_b) at A also causes a displacement at A′ of $0, \psi_b\,ds, -\psi_n\,ds$. Adding to these two displacements the change of displacement caused by the external loading given by the three first equations of (33), the total change of displacement between A and A′ is given by (36).

$$\left.\begin{aligned} dD_t &= D_n\frac{ds}{\rho} + \frac{T_t}{EA}ds. \\ dD_n &= -D_t\frac{ds}{\rho} + D_b\frac{ds}{\tau} + \psi_b ds + \frac{T_n}{GA'_n}ds. \\ dD_b &= -D_n\frac{ds}{\tau} - \psi_n ds + \frac{T_b}{GA'_b}ds. \end{aligned}\right\} \quad . \quad . \quad (36)$$

The effects of the shearing and axial forces can be proved to be negligible; these are omitted from equations (37).

$$\left.\begin{aligned} dD_t &= D_n\frac{ds}{\rho}. \\ dD_n &= -D_t\frac{ds}{\rho} + D_b\frac{ds}{\tau} + \psi_b\,ds. \\ dD_b &= -D_n\frac{ds}{\tau} - \psi_n\,ds. \end{aligned}\right\} \quad . \quad . \quad . \quad (37)$$

Equations (35) and (37) consist of differentials only; they are general and independent of the variable s.

After successive differentiations and substitutions, equations (35) may be reduced to a linear differential equation of the third order as in (38).

$$\left.\begin{aligned} &\rho\tau\frac{d^3\psi_t}{ds^3} + \left[\frac{d(\rho\tau)}{ds} + \tau\frac{d\rho}{ds}\right]\frac{d^2\psi_t}{ds^2} + \left[\frac{d\left(\tau\frac{d\rho}{ds}\right)}{ds} + \frac{\tau}{\rho} + \frac{\rho}{\tau}\right]\frac{d\psi_t}{ds} + \frac{d\left(\frac{\tau}{\rho}\right)}{ds}\psi_t \\ &\qquad = \frac{d\left[\tau\frac{d(\rho K_1\sigma M_t)}{ds}\right]}{ds} + \frac{d(\tau K_1 M_n)}{ds} + \frac{\rho}{\tau}K_1\sigma M_t + K_2 M_b. \end{aligned}\right\} \quad . \quad (38)$$

Solving equation (38) for ψ_t, and inserting this in equations (35), the values of ψ_n and ψ_b are found to be as in (39).

$$\left.\begin{aligned}\psi_n &= \rho\frac{d\psi_t}{ds} - \rho K_1\sigma M_t.\\ \psi_b &= \tau\left[\rho\frac{d^2\psi_t}{ds^2} + \frac{d\rho}{ds}\frac{d\psi_t}{ds} - \frac{d(\rho K_1\sigma M_t)}{ds}\right] + \frac{\tau}{\rho}\psi_t - \tau K_1 M_n.\end{aligned}\right\} \quad . \quad . \quad (39)$$

Similarly, from equations (37), the differential equation for the component of displacement D_t is

$$\rho\tau\frac{d^3D_t}{ds^3} + \left(\frac{d(\rho\tau)}{ds} + \tau\frac{d\rho}{ds}\right)\frac{d^2D_t}{ds^2} + \left[\frac{d\left(\tau\frac{d\rho}{ds}\right)}{ds} + \frac{\tau}{\rho} + \frac{\rho}{\tau}\right]\frac{dD_t}{ds} + \frac{d\left(\frac{\tau}{\rho}\right)}{ds}D_t = \frac{d(\tau\psi_b)}{ds} - \psi_n. \quad (40)$$

Solving this and inserting the value of D_t in equations (37), the values of D_n and D_b are found to be

$$D_n = \rho\frac{dD_t}{ds}. \qquad D_b = \tau\frac{d\left[\rho\frac{dD_t}{ds}\right]}{ds} + \frac{\tau}{\rho}D_t - \tau\psi_b. \quad . \quad . \quad . \quad (41)$$

Equations (10), (11), (12), (38), (39), (40), and (41) give the complete solution of a twisted curved beam subjected to a non-uniform load and a non-uniform bending moment.

Application of the General Equations to the Circular Helix.

Inserting the values of the radii of curvature and torsion from equation (16) in equations (38), (39), (40), and (41), the equations of deformation for circular helical beams are as in (42). .

$$\left.\begin{aligned}&\frac{a^2}{\sin^2\phi}\frac{d^3\psi_t}{ds^3} + \frac{d\psi_t}{ds} = K_1\sigma\frac{a^2}{\sin^2\phi}\frac{d^2M_t}{ds^2} + K_1 a\frac{dM_n}{ds} + K_1\sigma\cos^2\phi M_t\\ &\qquad + K_2\sin\phi\cos\phi\, M_b.\\ &\psi_n = \frac{a}{\sin^2\phi}\left(\frac{d\psi_t}{ds} - K_1\sigma M_t\right).\\ &\psi_b = \frac{1}{\sin\phi\cos\phi}\left[\frac{a^2}{\sin^2\phi}\left(\frac{d^2\psi_t}{ds^2} - K_1\sigma\frac{dM_t}{ds}\right) + \psi_t\sin^2\phi - K_1 a M_n\right]\\ &\frac{a^2}{\sin^2\phi}\cdot\frac{d^3D_t}{ds^3} + \frac{dD_t}{ds} = a\frac{d\psi_b}{ds} - \psi_n\sin\phi\cos\phi.\\ &D_n = \frac{a}{\sin^2\phi}\frac{dD_t}{ds}.\\ &D_b = \frac{1}{\sin\phi\cos\phi}\left[\frac{a^2}{\sin^2\phi}\frac{d^2D_t}{ds^2} + \sin^2\phi . D_t - a\psi_b\right].\end{aligned}\right\} \quad . \quad (42)$$

Changing the variable from s to the polar angle θ,

$$s = \frac{a}{\sin\phi}\theta,\ ds = \frac{a}{\sin\phi}d\theta,\ ds^2 = \frac{a^2}{\sin^2\phi}d\theta^2, \text{ and } ds^3 = \frac{a^3}{\sin^3\phi}d\theta^3.$$

Substituting these values in equations (42) we get (43).

$$\left.\begin{aligned}
&\text{(a)}\ \frac{d^3\psi_t}{d\theta^3}+\frac{d\psi_t}{d\theta}=\frac{K_1\sigma a}{\sin\phi}\frac{d^2M_t}{d\theta^2}+\frac{K_1\sigma a}{\sin\phi}\cos^2\phi.M_t+K_1a\frac{dM_n}{d\theta}+K_2a\cos\phi.M_b\\
&\text{(b)}\ \psi_n=\frac{1}{\sin\phi}\left(\frac{d\psi_t}{d\theta}-\frac{K_1\sigma a}{\sin\phi}M_t\right).\\
&\text{(c)}\ \psi_b=\frac{1}{\sin\phi\cos\phi}\left(\frac{d^2\psi_t}{d\theta^2}+\psi_t\sin^2\phi-\frac{K_1\sigma a}{\sin\phi}\frac{dM_t}{d\theta}-K_1aM_n\right).\\
&\text{(d)}\ \frac{d^3D_t}{d\theta^3}+\frac{dD_t}{d\theta}=a\left(\frac{d\psi_b}{d\theta}-\psi_n\cos\phi\right).\\
&\text{(e)}\ D_n=\frac{1}{\sin\phi}\frac{dD_t}{d\theta}.\\
&\text{(f)}\ D_b=\frac{1}{\sin\phi\cos\phi}\left(\frac{d^2D_t}{d\theta^2}+\sin^2\phi.D_t-a\psi_b\right).
\end{aligned}\right\}\quad(43)$$

Consider the staircase shown in *Fig.* 6, having a uniformly-distributed load of w lb. per foot and a uniformly-distributed bending moment of m ft.-lb. per foot of the helix, for which the values of T_t, T_n, T_b, M_t, M_n, and M_b are given by equations (24). Inserting these values in equation (43a) and rearranging, the following is obtained.

$$\frac{d^3\psi_t}{d\theta^3}+\frac{d\psi_t}{d\theta}=A_1+A_2\sin\theta+A_3\cos\theta+A_4\theta\sin\theta+A_5\theta\cos\theta+A_6\theta. \quad (44)$$

in which

$$A_1=-\frac{K_2a^2}{\cos\phi}C_1+\frac{a}{\sin\phi}(K_1\sigma\cos^2\phi+K_2\sin^2\phi)C_4,$$

$$A_2=-\frac{a^2\cos\phi}{\sin^2\phi}[2K_1(1+\sigma)+K_2]C_2-\frac{a}{\sin\phi}[K_1(1+\sigma\sin^2\phi)+K_2\cos^2\phi]C_5,$$

$$A_3=-\frac{a^2\cos\phi}{\sin^2\phi}[2K_1(1+\sigma)+K_2]C_3-\frac{a}{\sin\phi}[K_1(1+\sigma\sin^2\phi)+K_2\cos^2\phi]C_6,$$

$$A_4=+\frac{a^2\cos\phi}{\sin^2\phi}[K_1(1+\sigma\sin^2\phi)+K_2\cos^2\phi]C_3,$$

$$A_5=-\frac{a^2\cos\phi}{\sin^2\phi}[K_1(1+\sigma\sin^2\phi)+K_2\cos^2\phi]C_2,\quad\text{and}$$

$$A_6=+\frac{wa^3\cos^2\phi}{\sin\phi}[K_1\sigma-K_2].$$

Equation (44) is a linear differential equation of the third order with constant coefficients and a second member. The roots of the characteristic equation are $r_1=0$, $r_2=i$, and $r_3=-i$. Therefore if ψ_{t_1} is a particular solution of (44), the general solution is

$$\psi_t=C_7+C_8\sin\theta+C_9\cos\theta+\psi_{t_1} \quad . \quad . \quad . \quad (44a)$$

The particular solution is

$$\psi_{t_1} = A_1\theta - A_2\frac{\theta\sin\theta}{2} - A_3\frac{\theta\cos\theta}{2} + A_4\left(\frac{-\theta^2\sin\theta - 3\theta\cos\theta}{4}\right) + A_5\left(\frac{-\theta^2\cos\theta + 3\theta\sin\theta}{4}\right) + A_6\frac{\theta^2}{2}.$$

Therefore equations (44a), (43b) and (43c) become (45).

$$\left.\begin{aligned}
\psi_t &= C_7 + C_8\sin\theta + C_9\cos\theta + A_1\theta - A_2\frac{\theta\sin\theta}{2} - A_3\frac{\theta\cos\theta}{2} \\
&\quad + A_4\left(\frac{-\theta^2\sin\theta - 3\theta\cos\theta}{4}\right) + A_5\left(\frac{-\theta^2\cos\theta + 3\theta\sin\theta}{4}\right) + A_6\frac{\theta^2}{2}. \\
\psi_n &= \frac{1}{\sin\phi}\Bigg[C_8\cos\theta - C_9\sin\theta + A_1 - A_2\left(\frac{\theta\cos\theta + \sin\theta}{2}\right) \\
&\quad - A_3\left(\frac{-\theta\sin\theta + \cos\theta}{2}\right) + A_4\left(\frac{-\theta^2\cos\theta + \theta\sin\theta - 3\cos\theta}{4}\right) \\
&\quad + A_5\left(\frac{\theta^2\sin\theta + \theta\cos\theta + 3\sin\theta}{4}\right) + A_6\theta - \frac{K_1\sigma a}{\sin\phi}M_t\Bigg]. \\
\psi_b &= \frac{1}{\sin\phi\cos\phi}\Bigg[-C_8\sin\theta - C_9\cos\theta - A_2\left(\frac{2\cos\theta - \theta\sin\theta}{2}\right) \\
&\quad - A_3\left(\frac{-2\sin\theta - \theta\cos\theta}{2}\right) \\
&\quad + A_4\left(\frac{\theta^2\sin\theta - \theta\cos\theta + 4\sin\theta}{4}\right) \\
&\quad + A_5\left(\frac{\theta^2\cos\theta + \theta\sin\theta + 4\cos\theta}{4}\right) \\
&\quad + A_6 + \psi_t\sin^2\phi - K_1 a(1+\sigma)M_n\Bigg].
\end{aligned}\right\} \quad (45)$$

Equations (45) give the values of the angular rotation ψ_t, ψ_n, ψ_b at any point of the helix in terms of the function θ. Inserting in (43d) the values obtained from (45), and the values of the internal forces and moments T_t, T_n, T_b, M_t, M_n, M_b obtained from (24), and rearranging,

$$\frac{d^3D_t}{d\theta^3} + \frac{dD_t}{d\theta} = B_1 + B_2\sin\theta + B_3\cos\theta + B_4\theta\sin\theta + B_5\theta\cos\theta + B_6\theta + B_7\theta^2\cos\theta + B_8\theta^2\sin\theta. \quad (46)$$

in which $B_1 = \dfrac{a}{\sin\phi\cos\phi}\left[-A_1\cos 2\phi + \dfrac{K_1\sigma a\cos^2\phi}{\sin\phi}C_4\right]$,

$$B_2 = \frac{a}{\sin\phi\cos\phi}\Bigg[(1+\cos 2\phi)(C_9 - \tfrac{3}{4}A_5) + \left(\frac{3+\cos 2\phi}{2}\right)A_2 + \frac{K_1 a}{\sin\phi}(1+\sigma+\sigma\cos^2\phi)C_5 + \frac{2a^2K_1(1+\sigma)\cos\phi}{\sin^2\phi}C_2\Bigg],$$

$$B_3 = \frac{a}{\sin\phi\cos\phi}\left[(1+\cos 2\phi)(-C_8+\tfrac{3}{4}A_4)+\left(\frac{3+\cos 2\phi}{2}\right)A_3 + \frac{K_1 a}{\sin\phi}(1+\sigma+\sigma\cos^2\phi)C_6 + \frac{2a^2K_1(1+\sigma)\cos\phi}{\sin^2\phi}C_3\right],$$

$$B_4 = \frac{a}{\sin\phi\cos\phi}\left[-\left(\frac{1+\cos 2\phi}{2}\right)A_3+\frac{3-\cos 2\phi}{4}A_4 - \frac{a^2K_1\cos\phi}{\sin^2\phi}(1+\sigma+\sigma\cos^2\phi)C_3\right],$$

$$B_5 = \frac{a}{\sin\phi\cos\phi}\left[\left(\frac{1+\cos 2\phi}{2}\right)A_2+\frac{3-\cos 2\phi}{4}A_5 + \frac{a^2K_1\cos\phi}{\sin^2\phi}(1+\sigma+\sigma\cos^2\phi)C_2\right],$$

$$B_6 = \frac{a}{\sin\phi\cos\phi}\left[-A_6\cos 2\phi+\frac{wa^3K_1\sigma\cos^2\phi}{\sin\phi}\right],$$

$$B_7 = \frac{a}{\sin\phi\cos\phi}\left[\frac{1+\cos 2\phi}{4}A_4\right], \quad \text{and}$$

$$B_8 = -\frac{a}{\sin\phi\cos\phi}\left(\frac{1+\cos 2\phi}{4}A_5\right).$$

Equation (46) is a linear differential equation similar to (44). Therefore if D_{t_1} is a particular solution of (46), the general solution is

$$D_t = C_{10} + C_{11}\sin\theta + C_{12}\cos\theta + D_{t_1} \quad . \quad . \quad . \quad (46a)$$

The particular solution of (46) is

$$D_{t_1} = B_1\theta - B_2\frac{\theta\sin\theta}{2} - B_3\frac{\theta\cos\theta}{2} + B_4\left(\frac{-\theta^2\sin\theta-3\theta\cos\theta}{4}\right) + B_5\left(\frac{-\theta^2\cos\theta+3\theta\sin\theta}{4}\right) + B_6\frac{\theta^2}{2} + B_7\left(-\frac{\theta^3\cos\theta}{6}+\frac{3\theta^2\sin\theta}{4}+\frac{7\theta\cos\theta}{4}\right) + B_8\left(-\frac{\theta^3\sin\theta}{6}-\frac{3\theta^2\cos\theta}{4}+\frac{7\theta\sin\theta}{4}\right).$$

Equations (46a), (43e) and (43f) therefore become (47).

Equations (45), (47) and (24) represent the complete solution of the statically-indeterminate circular helical beam for the given loads. These equations have twelve constants which are found by inserting the conditions of restraint at the supports.

$$
\left.
\begin{aligned}
D_t &= C_{10} + C_{11}\sin\theta + C_{12}\cos\theta + B_1\theta - B_2\frac{\theta\sin\theta}{2} - B_3\frac{\theta\cos\theta}{2} \\
&\quad + B_4\left(\frac{-\theta^2\sin\theta - 3\theta\cos\theta}{4}\right) + B_5\left(\frac{-\theta^2\cos\theta + 3\theta\sin\theta}{4}\right) + B_6\frac{\theta^2}{2} \\
&\quad + B_7\left(-\frac{\theta^3\cos\theta}{6} + \frac{3\theta^2\sin\theta}{4} + \frac{7\theta\cos\theta}{4}\right) \\
&\quad + B_8\left(-\frac{\theta^3\sin\theta}{6} - \frac{3\theta^3\cos\theta}{4} + \frac{7\theta\sin\theta}{4}\right); \\
D_n &= \frac{1}{\sin\phi}\Bigg[C_{11}\cos\theta - C_{12}\sin\theta + B_1 - B_2\left(\frac{\theta\cos\theta + \sin\theta}{2}\right) \\
&\quad - B_3\left(\frac{-\theta\sin\theta + \cos\theta}{2}\right) + B_4\left(\frac{-\theta^2\cos\theta + \theta\sin\theta - 3\cos\theta}{4}\right) \\
&\quad + B_5\left(\frac{\theta^2\sin\theta + \theta\cos\theta + 3\sin\theta}{4}\right) + B_6\theta \\
&\quad + B_7\left(\frac{\theta^3\sin\theta}{6} + \frac{\theta^2\cos\theta}{4} - \frac{\theta\sin\theta}{4} + \frac{7\cos\theta}{4}\right) \\
&\quad + B_8\left(-\frac{\theta^3\cos\theta}{6} + \frac{\theta^2\sin\theta}{4} + \frac{\theta\cos\theta}{4} + \frac{7\sin\theta}{4}\right)\Bigg]; \\
D_b &= \frac{1}{\sin\phi\cos\phi}\Bigg[-C_{11}\sin\theta - C_{12}\cos\theta - B_2\left(\frac{-\theta\sin\theta + 2\cos\theta}{2}\right) \\
&\quad - B_3\left(\frac{-\theta\cos\theta - 2\sin\theta}{2}\right) + B_4\left(\frac{\theta^2\sin\theta - \theta\cos\theta + 4\sin\theta}{4}\right) \\
&\quad + B_5\left(\frac{\theta^2\cos\theta + \theta\sin\theta + 4\cos\theta}{4}\right) + B_6 \\
&\quad + B_7\left(\frac{\theta^3\cos\theta}{6} + \frac{\theta^2\sin\theta}{4} + \frac{\theta\cos\theta}{4} - 2\sin\theta\right) \\
&\quad + B_8\left(\frac{\theta^3\sin\theta}{6} - \frac{\theta^2\cos\theta}{4} + \frac{\theta\sin\theta}{4} + 2\cos\theta\right) + \sin^2\phi . D_t - a\psi_b\Bigg].
\end{aligned}
\right\} \tag{47}
$$

(*To be continued.*)

Specification for Road and Bridge Works.

THE Ministry of Transport and Civil Aviation has issued "Notes on the Second Edition of the Specification for Road and Bridge Works and on the preparation of Bills of Quantities" (H.M. Stationery Office. Price 3*s*. 6*d*.) as a supplement to the Ministry's Specification which was issued in 1957.

Some interesting notes are given on the use of lean concrete bases for flexible road surfaces. For example, it has been found that such concrete mixed in proportions richer than 1 part of cement to 15 parts of aggregate have on occasion resulted in serious cracks which may cause cracking of the topping of asphalt or coated macadam. For this reason the proportions have sometimes been reduced to 1 : 24.

Doubly-curved Roofs in Mexico.

SOME remarkable reinforced concrete structures (*Figs.* 1, 2 and 3) were described by Professor Felix Candela in a recent lecture arranged by the Cement and Concrete Association and given in London. Most of these were selected from more than one hundred and twenty buildings and other structures with doubly-curved roofs which have been erected in Mexico City during the past five years and for which Professor Candela was responsible for the structural design and construction. Earlier structures comprise domes, cylindrical and hyperbolic vaults, and conoids, but all recent structures have hyperbolic-paraboloidal roofs. Professor Candela stated that the word "shell" is often misused since many singly-curved structures such as barrel vaults are not true "shells". He defined a shell as a thin slab which can resist forces along the curvatures without bending, and condition for such action is that the surface must be curved in two directions. There are two fundamental types of doubly-curved shell, namely, the synclastic shell in which the curvature is always in the same direction (for example convex upwards as in a dome), and an anticlastic shell in which the curvature is convex upwards about one axis and concave about the other axis, that is the shape of a saddle. A synclastic shell is expensive to construct, since the centering is curved. Some anticlastic shells have the property that the surface is generated by straight lines.

Hyperbolic-paraboloidal surfaces are generated by two systems of straight lines (*Fig.* 4). Hyperbolic-paraboloidal roofs are therefore easy and economical to build, easy to design, and can have a variety of shapes.

The basic unit of many of the roofs is an "umbrella" formed by four contiguous anticlastic shells supported at the centre by a column. Such roofs are simple to construct because the shuttering is made of straight wooden strips, generally 2 in. wide and ¾ in. thick, fixed to straight bearers (*Fig.* 5). The surface of the concrete obtained by the use of narrow strips is commonly left exposed as a feature of the finish (*Fig.* 3). Doubly-curved structures are generally more economical in Mexico than those of conventional forms, one factor contributing to the cheapness being that the shuttering is used many times.

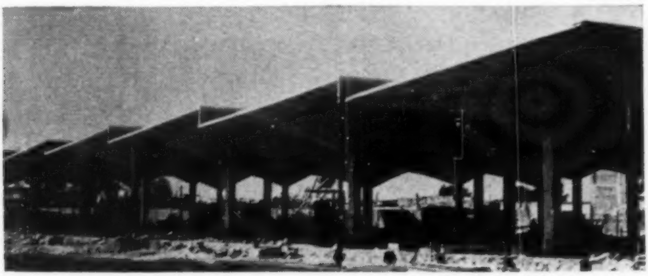


Fig. 1.—A Warehouse.

The roof of the warehouse shown in *Fig.* 1 comprises inverted umbrella units and covers about 1600 sq. ft. of floor. The slabs are 1½ in. thick. Since adjacent units in this type of construction are independent, they can be tilted so that the edges are separated and glazing of the vertical gap forms a north light as in *Fig.* 7 which gives details of the roof of a mill in Tlaxcala; the columns are at intervals of 44 ft. in one direction and 21 ft. in the other. The cost of this structure was about 5*s.* 6*d.* per square foot. Additional natural lighting can be provided by casting lenses in the slab as in *Fig.* 6.

Umbrella units are also used for houses, but in an inverted form, the apex and column being at the centre of the building. The roof is erected first and the rest of the construction proceeds under its protection.

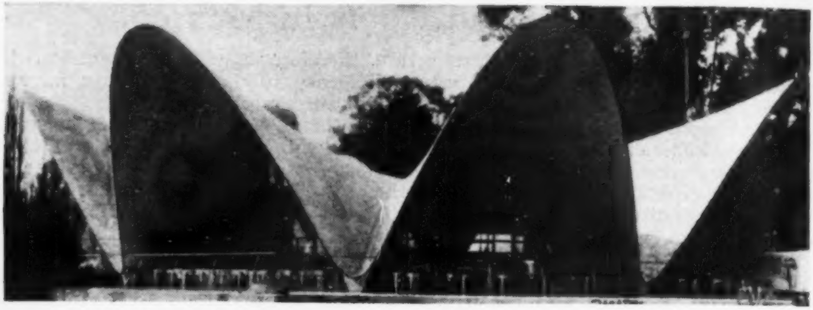


Fig. 2.—A Restaurant.

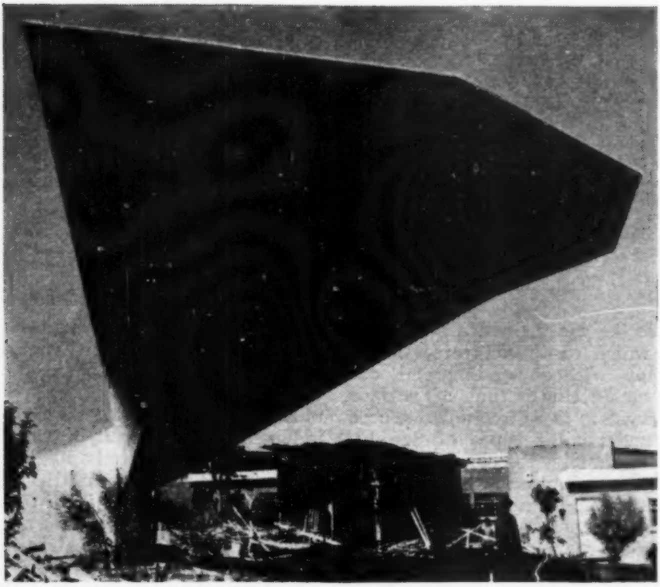


Fig. 3.—A Bandstand.

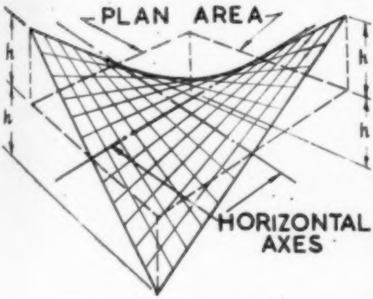


Fig. 4.—Hyperbolic Paraboloid formed by Rectilinear Generators.

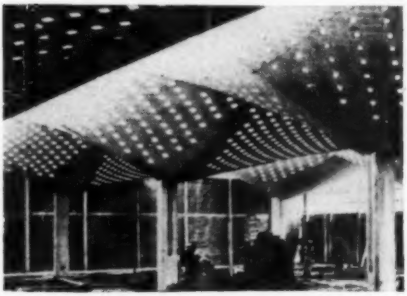


Fig. 6.—A Factory.

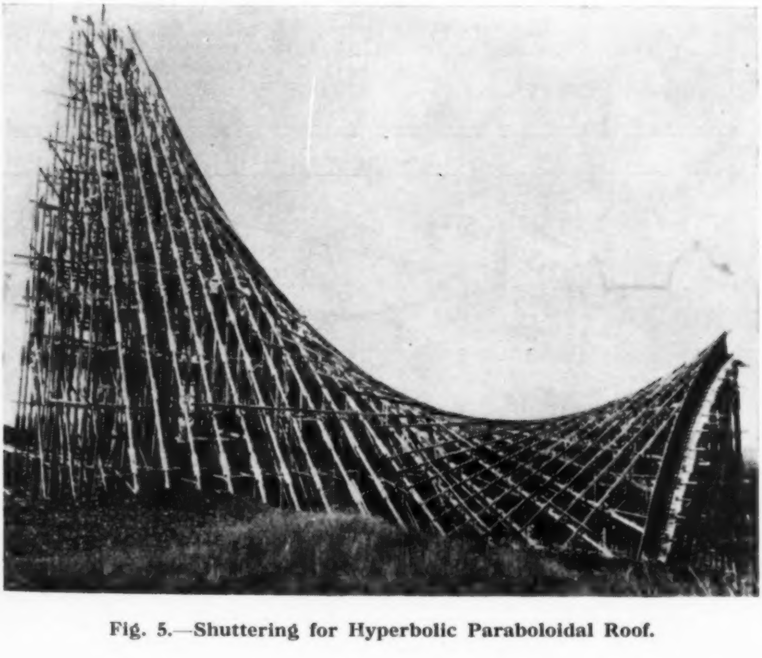
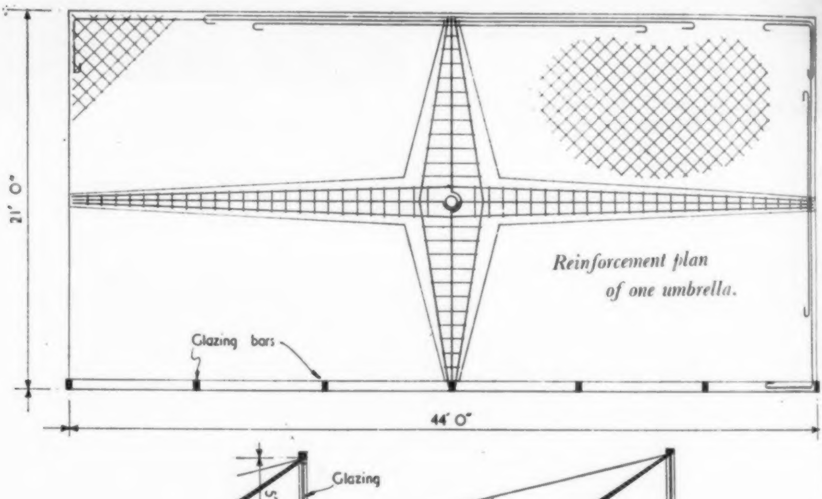


Fig. 5.—Shuttering for Hyperbolic Paraboloidal Roof.

The ground at Mexico City is a water-bearing silt and considerable subsidence, up to 2 ft. or so in a year, of the foundations of buildings may occur. Therefore an unusual column-base comprising a large umbrella unit is frequently used. Such bases are provided for the building in *Fig*. 7. Similar bases 10 ft. by 11 ft. 6 in. and comprising 6-in. slabs are used for the Customs sheds seen in *Fig*. 8 where the ground was excavated to the shape required and a layer of lean mortar was applied to the surface on which the base was cast.

The sheds in *Fig*. 8, the roofs of which are an example of an early cylindrical vault, comprise two buildings each 720 ft. long and one 230 ft. long. The roof slabs are 1½ in. thick and the vault is 66 ft. wide. The canopies project 20 ft. and



Reinforcement plan of one umbrella.

Section through one umbrella and column showing the umbrella-type foundation.

Fig. 7.—A Mill Building.

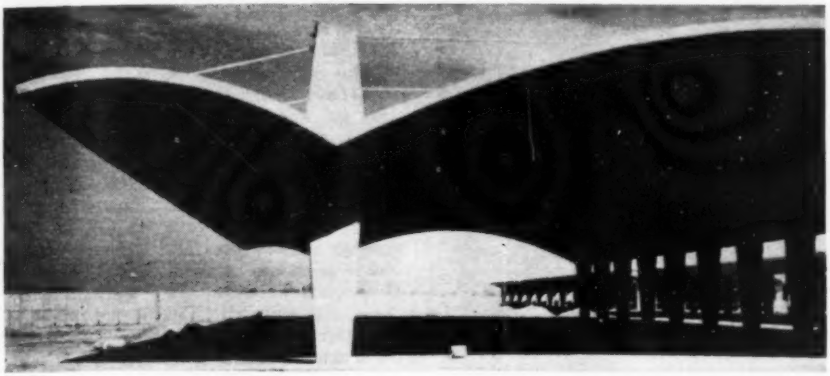


Fig. 8.—Customs Sheds.

are tied by rods connected to extensions of the columns above the roof. No beams are provided at the valleys since the vaults are designed to span between the columns, which are at 35 ft. centres.

The canopy of a bandstand (*Fig.* 3) comprises six hyperbolic-paraboloidal slabs which cantilever 40 ft. from the support and is an example of the use of this shape in an extreme form. The length along the free end of the canopy is 60 ft. In the design the structure was assumed to act as a series of anticlastic slabs; consideration of the structure as a simple cantilever produced similar results.

Another example of hyperbolic-paraboloidal vaults is the roof of a restaurant at Xochimilco (*Fig.* 2), which is formed

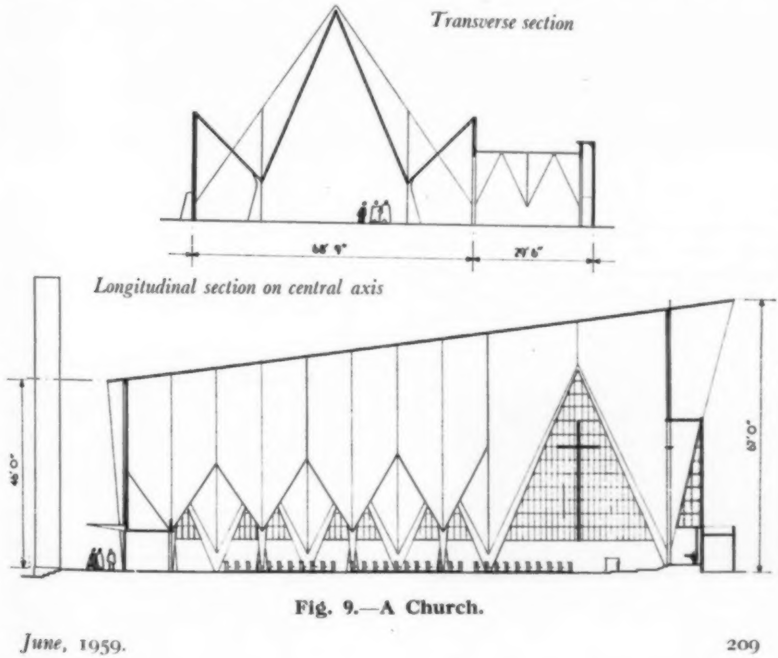


Fig. 9.—A Church.

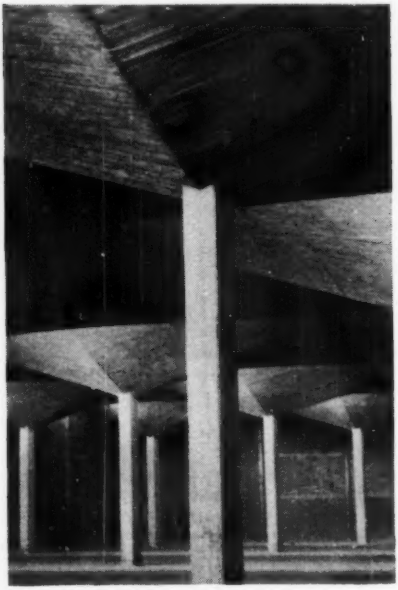


Fig. 10.—A Warehouse Roof.

by six vaults $1\frac{1}{2}$ in. thick intersecting at the centre of the building. A similar roof also $1\frac{1}{2}$ in. thick, covering the hall of the Stock Exchange in Mexico City, is formed of four such vaults covering an area 46 ft. by 83 ft. 6 in. There is no stress in the edges of the vaults, which in this building are inclined inwards. A small laboratory for the study of cosmic rays has a doubly-curved roof which is only $\frac{5}{8}$ in. thick in order to obstruct the passage of the rays as little as possible.

The Church of La Virgen Milagrosa (*Fig.* 9) in Mexico City is an example of the use of the umbrella unit in an extreme form which, in this case of a monumental building, is not necessarily the cheapest. The slabs of roofs of the nave and side chapels are $1\frac{1}{2}$ in. thick throughout.

Professor Candela is Professor of Design in the Faculty of Architecture at the National University of Mexico, and was also the contractor for the structures described in the foregoing.

A Large Water Tank.

After considering designs for reinforced and prestressed circular tanks and for a rectangular tank, with a capacity of about 2,700,000 cu. ft., the square reinforced concrete structure shown in *Fig.* 1 has been built at Helsinki, Finland. The tank is 190 ft. square, and is 50 ft. above ground level so that the space beneath it may be used for offices, garages, and storage purposes. The walls (*Fig.* 2) are 7 in. thick and are in the form of arches of about 18 ft. span which are connected to the roof. The concrete was placed continuously during five days. The foregoing is abstracted from "Nordisk Betong", No. 3, 1958.

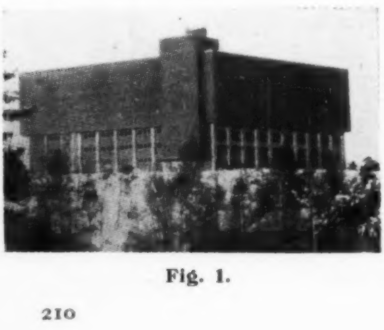


Fig. 1.

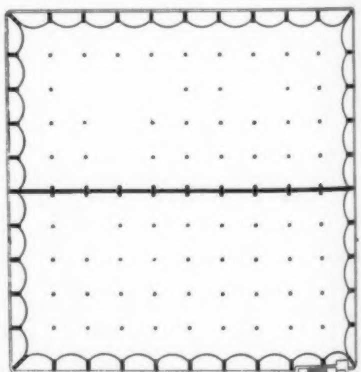


Fig. 2.

Concrete Construction at Berkeley Nuclear Power Station.

THE civil engineering work at the nuclear power station in course of construction at Berkeley, Gloucestershire, is now almost complete and includes more than 100,000 cu. yd. of concrete in the two reactors, 20,000 cu. yd. in other works in the nuclear area, 48,000 cu. yd. in the turbine hall and other conventional structures, and 22,000 cu. yd. in the cooling-water system. *Fig.* 1 is a view of the works early this year. The two reactors are near the middle of the photograph, and the turbine hall is behind them. The main concrete plant, reinforcement store, and precasting yard are in front of the reactors. The outfall works of the cooling-water system are to the left; the intake works are in the River Severn beyond the left-hand side of the illustration.

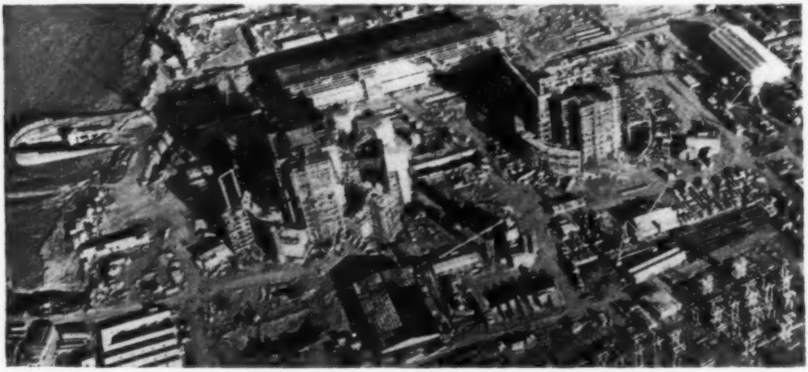


Fig. 1.—View of Site.

Concrete Plant for Nuclear Area.

A preliminary description of the concrete batching, mixing, and distributing plant for the reactors and other structures in the nuclear area was given in our number for January, 1958. This plant, which had a capacity of 80 cu. yd. per hour but is no longer in use, comprised four steel silos each containing 425 tons of cement, two 2-cu. yd. mixers, and four 6-in. concrete pumps. The aggregates, which are mainly crushed limestone and siliceous sand, were loaded into the hoppers of the batching plant by inclined conveyors. The size of aggregate to be delivered was selected by remote control from the top of the batching plant.

The concrete was discharged from the mixers directly into the pumps, from which four pipes were laid to a position between the two reactors so that concrete could be pumped to either structure by rearrangement of the end sections of the pipes. The pipes passed under the intervening road in a temporary corrugated-steel culvert of 7 ft. diameter. With this installation the concrete was pumped a distance of 550 ft. and to a height of 50 ft. For placing concrete at greater heights two pumps were installed near each reactor and the concrete was then pumped a distance of 200 ft. and to a height of 85 ft., although this was not the limiting distance. The concrete was brought from the mixers in end-tipping lorries which discharged from a platform into remixing hoppers at the pumps in the new position. Access to the platform, which was above the pumps, was by means of a Bailey-bridge ramp.

For the construction of the reactors above 85 ft. it was more convenient to distribute the concrete in bottom-opening skips of 2 cu. yd. capacity lifted by tower cranes, two of which are installed at each

of the two reactors (*Fig.* 2). The towers are 130 ft. high and the cranes are capable of lifting 15 tons at a radius of 90 ft. or 6 tons at the maximum working radius of 125 ft.

Before the main concrete plant was installed, concrete for temporary works and for "blinding" the foundation for the first reactor was mixed in six 14/10 mixers placed at various positions. Now that the main concreting plant is not in use, the plant for the small amount of work remaining comprises two cement silos (each of 20 tons capacity), batch-weighers, and two 18/12 mixers, and the concrete is distributed in high-discharge lorries, from which it is discharged into skips or the remixing hopper of a concrete pump.

The foundations of the reactors are 35 ft. below ground, and each area of about 300 ft. diameter is surrounded by a retaining wall which was concreted in two lifts each about 17 ft. high. Each of the two steel pressure-vessels is on a reinforced concrete raft 14 ft. thick and 150 ft. diameter and supporting 50,000 tons. The rafts were cast in bays each containing 70 cu. yd. to 190 cu. yd. of concrete. The shield-wall around each vessel is 8 ft. 6 in. thick (*Fig.* 3) and is of concrete of high quality cast in lifts of 3 ft. each in timber cantilevered shuttering (*Fig.* 4). Steel shutters made on the site were used for the walls of the adjacent blower pits, the concrete in which was placed in one lift of 13 ft. Extra protection is provided by additional walls 4 ft. thick where the gas-ducts from the boilers pass through the shield; at the bottom of the reactor these walls are 20 ft. high and were cast in one lift. The concrete in these deep lifts was compacted by immersion vibrators.

The shield over the vessel is 12 ft. thick and is penetrated by the tubes for the fuel-elements and control-rods. The concrete in the top shield is placed in successive layers. Steel girders are embedded in the bottom layer to strengthen it to carry the weight of concrete in the upper layers.

The concrete in the shield walls and top shield is high-quality limestone concrete, but in the top shield there is a 7-in. layer of concrete having iron shot as aggregate.

Fig. 2.—Reactor in Course of Construction.

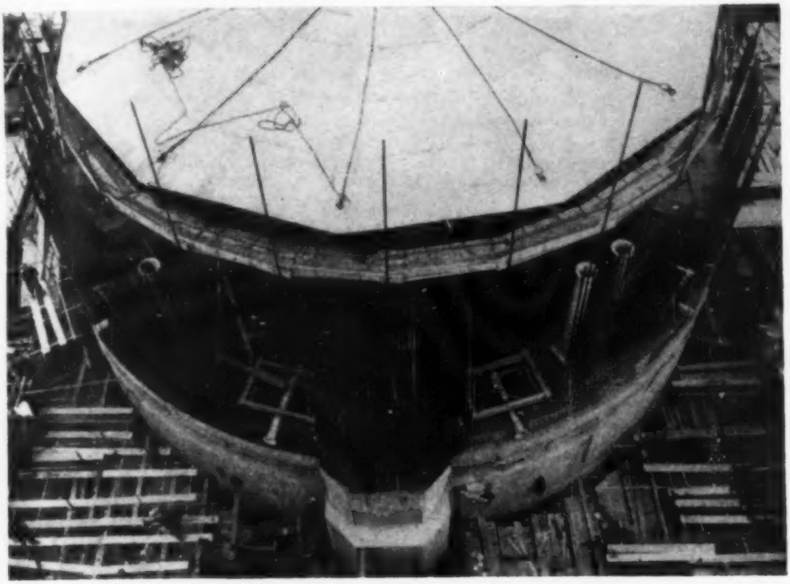


Fig. 3.—Shield Wall during Construction.

Cores cut from the concrete in the shield wall had crushing strengths of 8600 lb. per square inch at one year; the aggregate-cement ratio of this concrete is 6·5 and the water-cement ratio is 0·6. Tests made in the laboratory on the site indicate that concrete with mixed iron shot of No. 14-gauge and No. 7-gauge as aggregate had a density of 340 lb. per cubic foot.

Precast Panels.

Many precast units of various types for all parts of the contract are made in a precasting yard (*Fig.* 5) which has a separate batching plant and a 10/7 mixer for special concrete.

A store (seen to the left in *Fig.* 5) is provided for lightweight aggregates and special aggregate for facings. A 7-tons jib-crane travelling on rail tracks serves the yard. The products made include 1030 panels for the flat roofs of the blower houses and about 2000 wall slabs.

The roof panels are plain on top and recessed on the underside, and are cast top uppermost in the open. The sides of the moulds are steel road forms, and the bottoms are of concrete shaped to form the recesses. Pipes and vents embedded in the bottom provide for the release of the casting from the moulds by air pressure. The panels are lifted by the crane with slings attached to four eyes cast in the concrete (*Fig.* 6). The largest panels are 19 ft. 9 in. long, 6 ft. 6 in. wide, and 9 in. thick. Three panels were made each week in warm weather and two in cold weather in each mould.

The wall slabs are generally 15 ft. 6 in. long and 5 ft. 2 in. or 3 ft. 6 in. wide, or 2 ft. 6 in. by 3 ft. 6 in. The largest slab weighs 2¼ tons.

The slabs are recessed on the inner face and have an exposed-aggregate outer face. They are cast under cover on the floor of the shed at the casting yard. The shed is about 50 ft. wide and contains two casting lines over each of which is a travelling crane of 18 ft. span. Each crane can lift the equivalent of two loads of 30 cwt. each. The slabs are cast in hardwood moulds with resin-bonded plywood sheeting attached to welded steel bearers. The recesses are formed by timber blocks. If the slabs are not to be backed by concrete cast in place after erection, precast blocks of foamed-slag

concrete are placed in the moulds in place of the timber blocks and form part of the slab.

A layer of backing concrete 4 in. thick is deposited in the bottom of the mould. The facing concrete containing crushed stone aggregate is then placed in a layer 2 in. thick. Three to four hours after casting, the face of the slab is watered and brushed with a bass broom to remove the mortar and expose the aggregate. The water and sludge drain into channels provided along the outer edge of each of the casting lines.

The shutters on the sides are removed the day after casting, and the slab is tilted to a position nearly vertical by means of two lifting-blocks suspended from the crane gantry. Another pair of blocks is attached to lifting-eyes cast in the top edge of the slab, which is then lifted, together with the back shutter, off the steel bearers. The bearers are lowered on to the floor and the back shutter is removed from the suspended slab. The crane, with the slab suspended from it, is pushed out of the building and the slab is stacked temporarily in a vertical position against a timber frame in the open. The slab is later transferred to the main stacking yard by the derrick crane and stored in a nearly vertical position. Each mould is used once daily and the slabs are removed from the moulds and stacked temporarily about twenty hours after casting.

Cooling-water System.

The cooling-water system comprises the intake works, which are about 500 ft. offshore; two tunnels each about 900 ft. long for conveying the water from the intake

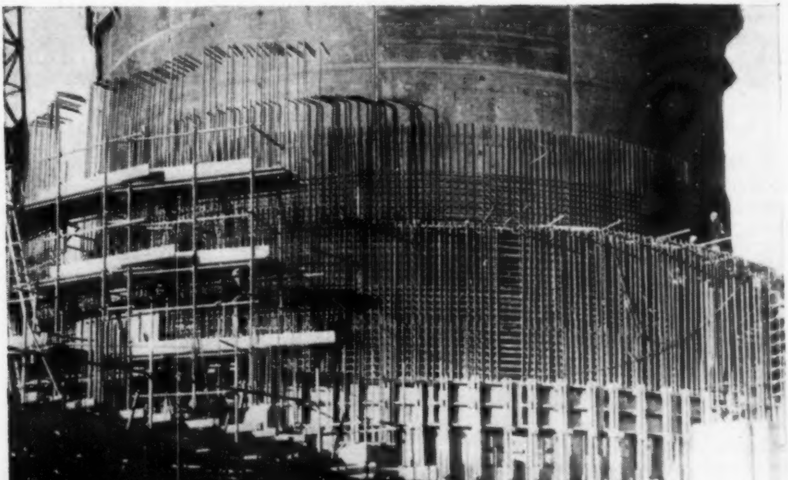


Fig. 4.—Shuttering and Reinforcement of Shield Wall.

Fig. 5.—The Precasting Yard.

to the fore-bay of the pump-house; a man-access tunnel about 650 ft. long from the intake to the shore; the pump-house; culverts from the pump-house to the turbine hall and thence to the outfall works. To prevent re-circulation of warm water, a steel sheet-pile baffle-wall about 1500 ft. long is provided immediately behind the intake and in front of the outfall.

A separate concrete batching and mixing plant is provided to supply the concrete for the cooling-water system and the other conventional works. This plant comprises a cement silo of 300 tons capacity and two 1-cu. yd. mixers. Aggregate is delivered to the hoppers of the batching plant by a Scotch derrick crane of 7 tons capacity. The concrete is distributed in 1-cu. yd. skips (with roller-type gates) carried on lorries, or by two pumps. An air-entraining agent is used; in addition to improving workability it prevents the pipes from the pumps becoming choked.

The coarse aggregate is crushed limestone graded from $\frac{3}{4}$ in. to $\frac{3}{8}$ in. For concrete for various purposes the proportions (by weight) are 1 : 1·9 : 3·5, 1 : 2·15 : 3·85, 1 : 2·5 : 4·5, and 1 : 3 : 5 for nominal crushing strengths at twenty-eight days of 4500, 3000, 2250, 1500 lb. per square inch respectively. The water-cement ratio (excluding moisture in the aggregates) is 0·54, 0·60, 0·75, and 0·80 respectively. The compaction-factor was 0·94 for all mixtures.

When constructing the substructure of the pump-house the concrete was placed in lifts of 6 ft., the volume placed continuously being up to 400 cu. yd. in one day. Vertical construction and expansion joints were formed in the walls by inserting in the concrete inflatable rubber tubes of 4 in. diameter. After the concrete had set, the tubes were deflated and shrinking of the concrete resulted in hair-cracks around the holes formed by the tubes. The holes were filled with an expanding grout to seal the cracks.

The tunnels were constructed from shafts 10 ft. diameter and 100 ft. deep sunk in the bank of the river, and compressed air was used during part of the driving. The tunnels are lined with bolted precast concrete segments to which 2 in. of gunite was applied, resulting in

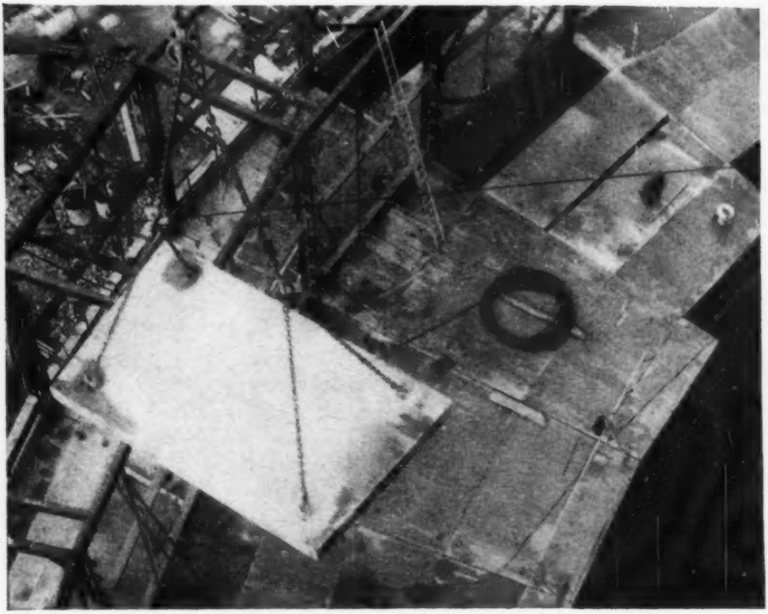


Fig. 6.—Erecting Precast Roof Panels.

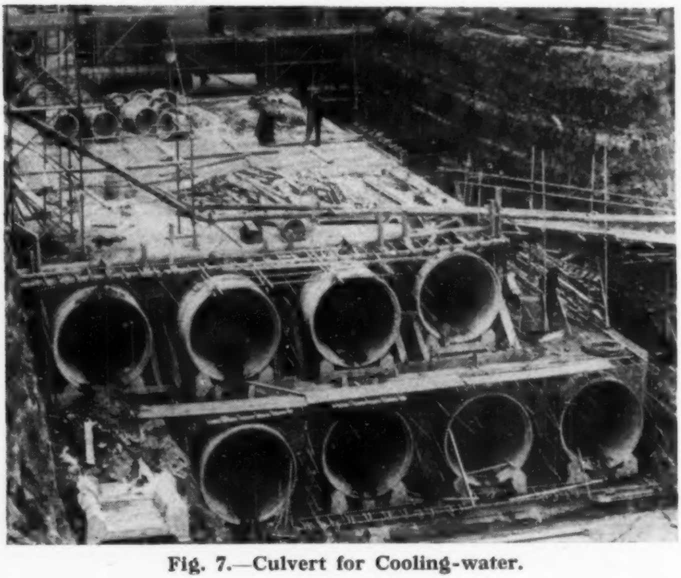


Fig. 7.—Culvert for Cooling-water.

internal diameters of 9 ft. 6 in. and 7 ft. for the water tunnels and man-access tunnels respectively. A lift-shaft is provided at both ends of the access tunnel. The two intake works are each being constructed within a circular cofferdam of 62 ft. 6 in. diameter formed with steel piles of hollow square cross section, and the shaft to the man-access tunnel within a cofferdam of 24 ft. diameter. Owing to the extremely wet conditions, the on-shore shafts from the water tunnels to the forebay of the pump-house are lined with precast slabs cast on the site. Each slab weighs 7 tons and the space between the slabs and the excavation is filled with concrete.

The connection to and from the turbine hall is a reinforced concrete culvert 40 ft. wide and 20 ft. deep overall and contains four inlet pipes in an upper tier and four outlet pipes in a lower tier (*Fig.* 7). The spun-concrete pipes, of which there are 3000 ft., are 6 ft. 9 in. diameter.

The outfall works, which was constructed within a horseshoe cofferdam, comprises a reinforced concrete headwall beyond which the cooling water will flow out on to an apron and then discharge into the river.

The station is being constructed by A.E.I.-John Thompson Nuclear Energy Co., Ltd., and the consulting engineers are Messrs. W. S. Atkins & Partners. The civil engineering contractors for the reactors and ancillary buildings, cooling ponds, active-waste buildings, and other work in the nuclear area are Messrs. John Laing & Son, Ltd., who also operate the precasting yard. Messrs. Balfour Beatty & Co., Ltd., are the civil engineering contractors for the cooling-water system, the turbine hall, and other conventional structures.

FIFTY YEARS AGO.

From "CONCRETE AND CONSTRUCTIONAL ENGINEERING", June–July, 1909.*

THIS bridge, near Teufen, Switzerland, has a span of 259 ft. and a rise of 87 ft. The approaches comprise six arches of 33 ft. 7 in. span. The road is 216 ft. above the river and is 22 ft. 7 in. wide. The main span is fixed at the ends. The thickness of the arch is 4 ft. at the crown and 7 ft. at the abutments. The construction was started in March, 1907, and the bridge was opened to traffic in November, 1908. The concrete was mixed in the proportions of 1 cu. yd. of coarse aggregate, ½ cu. yd. of sand, and 600 lb. of cement (4 : 2 : 1 by volume), and had a compressive strength of 4330 lb. per square inch at 28 days.

* "Concrete and Constructional Engineering" appeared in alternate months until September, 1909.

The late Mr. F. E. Wentworth-Sheilds.

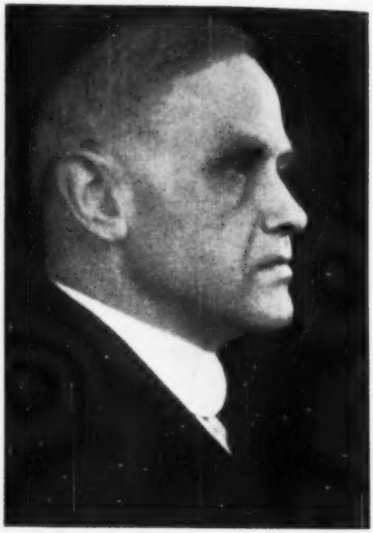
In our last number we reported that the University of Southampton had decided to award an honorary doctorate of science to Mr. F. E. Wentworth-Sheilds on July 3 next. Alas, he died on May 9 and missed the pleasure of receiving this honour from the city in which he had spent most of his life and which had benefited so much from his work.

Mr. Wentworth-Sheilds, who was in his ninetieth year, started work as a civil engineer in 1888, and after experience in the construction of canals, docks, dams, and railways at home and abroad went to Southampton in 1901 where he worked on various docks and quays until in 1907 he was appointed Docks Engineer of the London and South Western Railway and continued in this post when the S.W. Railway was merged with the Southern Railway until his retirement in 1936. During this period he was responsible for the vast extension of the facilities at Southampton Docks, including the Ocean Dock, Trafalgar Dock, the provision of eight berths for ocean liners on reclaimed land, and the King George V graving dock. For his services to the development of the port of Southampton he was awarded an O.B.E.

He became an Associate Member of the Institution of Civil Engineers in 1895 and a full member in 1905, and was elected President in 1944. He was one of the earliest members of the Concrete Institute (now the Institution of Structural Engineers), and was a pioneer in the use of reinforced concrete in this country; so long ago as 1906 he was publicly deprecating the use of the very wet mixtures then commonly used to enable concrete to be poured between closely-spaced reinforcement bars.

Mr. Wentworth-Sheilds was one of the friendliest as well as one of the ablest engineers of his time, and gave unstintedly of his time in imparting his knowledge to others. His election to membership of the Athenæum Club gave him much pleasure —indeed he said at the time that it was the greatest honour he had received.

His many friends will be happy to know that he preserved all his faculties and was fully active until a few days before his death; indeed only a week or two before



his passing the writer was handling some notes he had written for the revision of " Reinforced Concrete Piling ", a book he had written in collaboration with the late W. S. Gray, and was amazed at the firmness of the handwriting and the clarity of his thoughts.

Protection against Nuclear Radiation.

Seven papers on the production of concrete for use as a shield against nuclear radiation and which have been published in the Journal of the American Concrete Institute in recent years are now available in book form. Some of the papers deal with the properties required in such concrete, and others describe the methods used in the U.S.A. for producing heavy concrete with special aggregate. Copies are available from the Institute, P.O. Box 4754, Redford Station, Detroit 19, Michigan, U.S.A., at a price of $4. (Payment from Great Britain must be made by a draft obtainable from the customer's bank.)

Railway Reconstruction at Barking.

PRESTRESSED PRECAST BEAMS.

REINFORCED and prestressed concrete are being used extensively in civil engineering works required in connection with the electrification and other improvements of British Railways in east London and the adjacent Home Counties. One of the major works is the rearrangement of the railway tracks at the junctions at Barking to obviate trains having to cross other lines on the same level when passing from one route to another. To the west of Barking station the line from St. Pancras to Tilbury is being carried over the main line from Fenchurch Street to Southend and the London Transport line. The track used by London Transport trains from Upminster is being diverted to pass under the main line east of Barking station and over the main line west of the station. The decks of the approach viaducts and bridges and the roof of the underpass are of composite prestressed concrete construction.

Other works include a reinforced concrete subway under the station ; a road bridge of 30 ft. span with a deck of composite prestressed concrete; a steel bridge with a reinforced concrete deck over the river Roding, comprising three spans each 56 ft. long carried on massive plain concrete abutments; a steel footbridge having a maximum span of about 95 ft. and carried on precast reinforced concrete piers; and several concrete retaining walls, the lowest of which are of precast units. To the west of the station the new structures are carried on cast-in-place expanded piles. Metallurgical supersulphated cement is used for the piles and all concrete below ground because of the presence of contaminated ground-water; and for part of the superstructures because of the sulphurous fumes from locomotives.

Fig. 1.—General View.

Viaducts and Bridges.

The two bridges over the tracks and the approaches (*Fig.* 1) are of similar construction in so far as where the tracks are only a few feet above the ground concrete retaining walls are provided (as seen in the background of *Fig.* 2) and the tracks are on earth filling. The bases of opposite walls are connected by underground ties comprising two rails encased in concrete. Old rails are also used as the main reinforcement in the walls, but the secondary reinforcement is mild steel bars. Where the tracks are at a higher level, concrete piers (*Fig.* 2) are provided to support a deck of composite prestressed concrete construction which spans longitudinally (*Fig.* 3); in both constructions there are sixteen approach spans of 40 ft. each. The piers are lightly reinforced and are carried on piles.

Where the elevated tracks cross other tracks, reinforced concrete walls and columns supporting longitudinal beams (*Figs.* 4 and 5) cast in place are provided to carry the deck. There are three such spans over other tracks to carry the Tilbury line and one to carry the London Transport line. The walls and columns are so arranged that nowhere is there a wall on both sides of a track.

The complexity produced by the crossings over tracks on the skew is overcome by constructing triangular areas of the deck in reinforced concrete cast in place

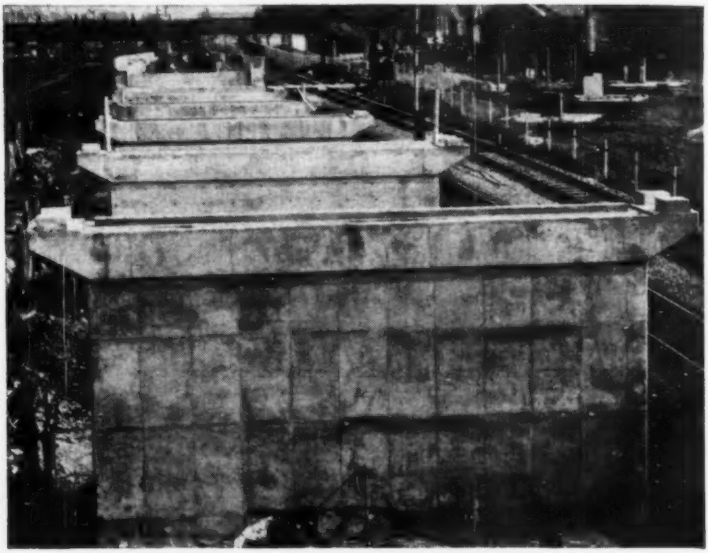


Fig. 2.—Piers for an Approach.

(*Fig.* 7). Where practicable, the arrangement of the prestressed deck spanning longitudinally is maintained. Elsewhere the prestressed deck spans transversely (*Fig.* 5) between the walls and longitudinal beams, which are designed as continuous reinforced concrete beams. As the beams serve as parapets, they are of greater depth than would otherwise be required and therefore do not require much reinforcement (*Fig.* 4). To ensure that little or no bending moment is produced on the columns, bearings as shown in *Fig.* 8 are provided. The sliding

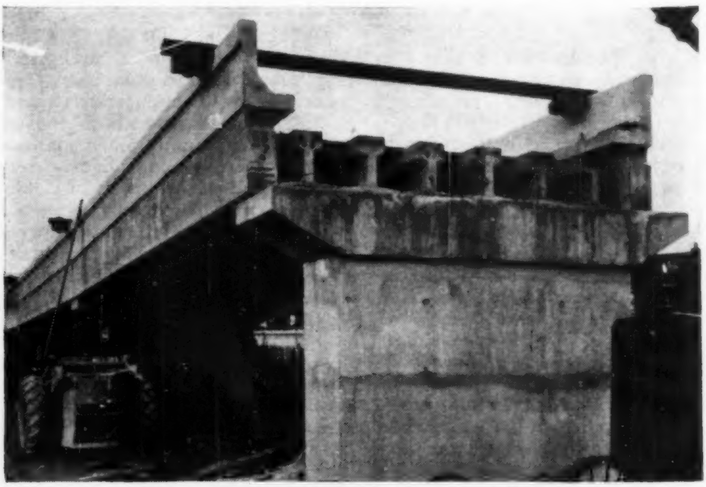


Fig. 3.—Approach during Construction.

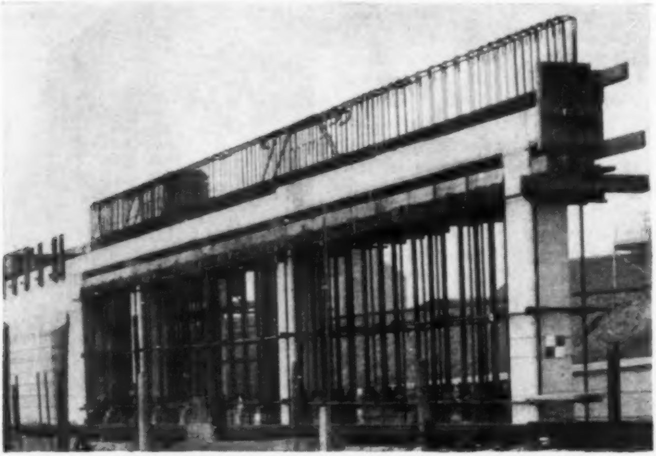


Fig. 4.—Longitudinal Beams during Construction.

bearing at (a) is for end supports and the pinned bearing at (b) is for intermediate supports.

The ordinary composite deck construction comprises precast prestressed beams of inverted tee-section with concrete cast in place between and over the webs. The designed span of the beams in the approach spans is 38 ft.; they are designed to be freely-supported members capable of carrying the weight of the wet concrete and constructional loads. They are 30 in. deep and the bottom flange is 27 in. wide. (Beams 15 in. to 21 in. deep are provided for the smaller transverse spans.) When the deck is completed the overall thickness

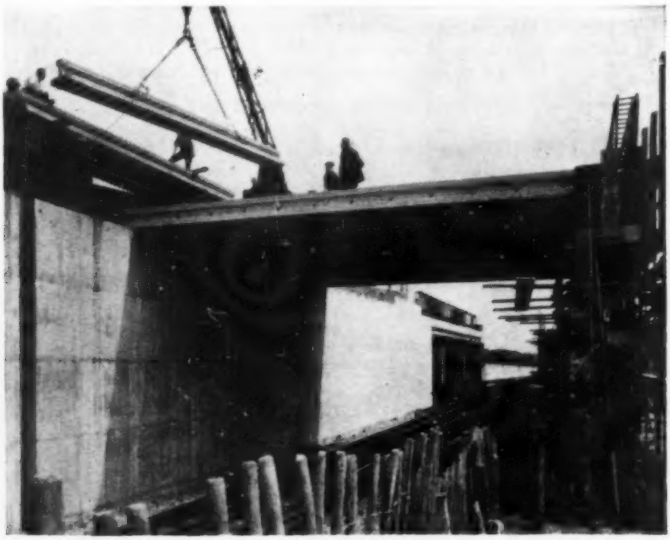


Fig. 5.—Bridge with Beams Spanning Transversely.

will be 36 in.; the composite construction will be capable of carrying the dead loads and the traffic loads. In this condition the beams act as partially-prestressed members, there being a limited tensile stress in the concrete under the greatest live load. Reinforcement spanning transversely to the beams is provided in the top and near the bottom of the deck, the latter passing through holes in the webs. The two layers are connected by vertical links (*Figs.* 7 and 9). This method of construction enables a relatively thin deck to be provided which is economical to construct since the weights of the beams are small and no shuttering is required for the soffit of the deck. The decks are coated with bituminous felt upon which are laid tiles set in bitumen. The purpose of the tiles is to prevent the felt being perforated by the ballast or the chippings which cover the deck alongside the tracks.

Fig. 6.—Erecting a Beam.

Casting the Prestressed Beams.

The beams are made by the pre-tensioning method on beds such as those in *Fig.* 10, where each of the three beds has a nominal length of 140 ft. and each has a prestressing capacity of 200 tons. The wires are 0·276 in. diameter and are tensioned in pairs after being straightened by applying an initial tension of 400 lb. in each wire. Before leaving the concrete works, some beams are tested in a rig of 50 tons capacity (*Fig.* 11). The rig is unusual in so far as the beam is placed over the rig instead of between the frames of the rig. The beam is placed on the rig by a 10-tons steam crane, by which, it is thought, the beam can be placed more gently than by other means. The load is applied by a jack which thrusts downwardly on a steel

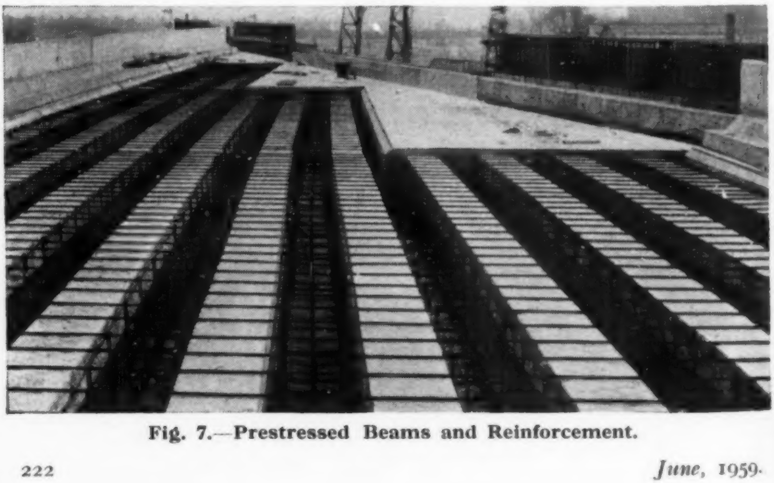


Fig. 7.—Prestressed Beams and Reinforcement.

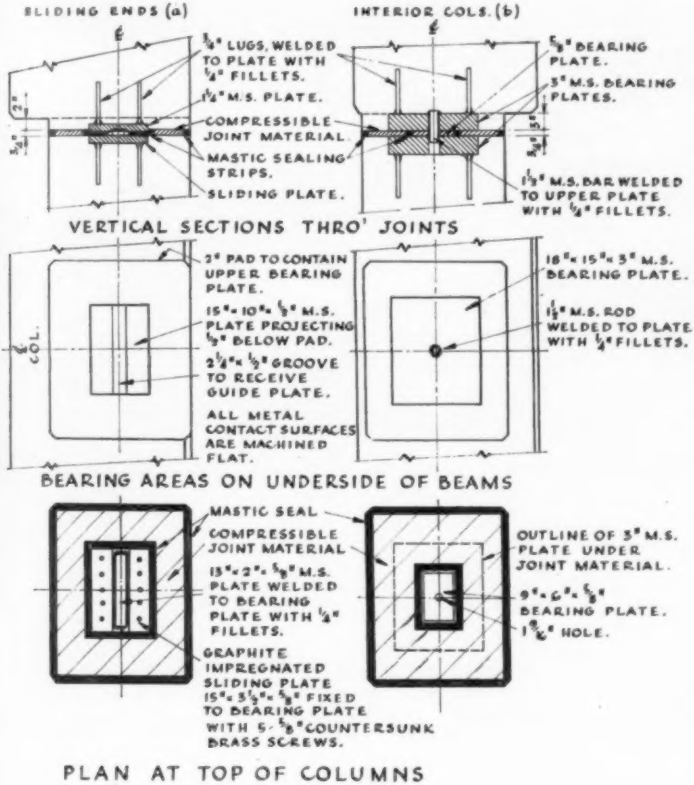


Fig. 8.—Details of Bearings.

beam suspended from the beam being tested. The deflections of the supports at the ends of the beam are measured on spring-loaded gauges; similar readings are taken at midspan where there is a proving ring.

Erecting the Prestressed Beams.

Two methods of erecting the beams on the site are shown in *Figs.* 5 and 6. In

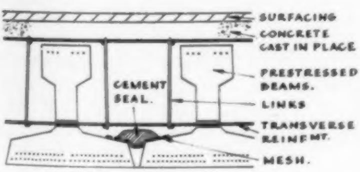


Fig. 9.—Composite Deck.

the former the method is the ordinary two-point lift by crane. The method shown in *Fig.* 6 is used when a lorry with lifting-gear attached can travel on beams already in position. In this case the beam, in a steel cradle, is carried by the lorry from the storage ground and lowered directly into position. The bearing of the beams on the piers comprises a strip of mild steel, ⅝ in. thick, 11 in. wide, and 23 ft. long, secured to the top of the pier. A corresponding plate 6 in. wide and of breadth equal to that of the flange of the beam is built into the soffit of the beam. During erection there is a tendency for the beams to creep down the slope due to vibration of passing trains. This tendency is counteracted by providing temporary inclined timber shoring at the lower end of each span.

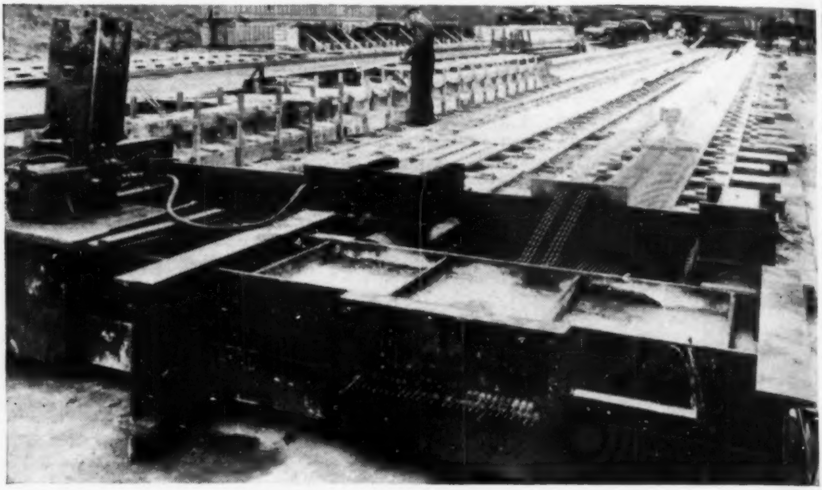


Fig. 10.—Stressing Bed for Beams.

When the beams are in position, the gap between adjacent bottom flanges is sealed. If the gap is narrow it is covered by a plug of sulphate-resistant cement, but if it is wide a layer of light galvanised-wire mesh is placed over the gap, as in *Fig.* 9, before the cement-plug is placed.

The construction of the edge-beams of the approach spans is seen in *Fig.* 3, and comprises a rectangular prestressed beam surmounted by a precast ell-shape parapet.

The Under-pass.

The under-pass is in general an open reinforced concrete channel, the walls of which are cantilevers except that, where it is deep enough to allow passage of the trains, 15-in. by 8-in. precast struts are provided between the tops of the wall. Where tracks pass over the under-pass a composite roof is provided similar to the decks already described. The under-pass is mainly in water-bearing sand and

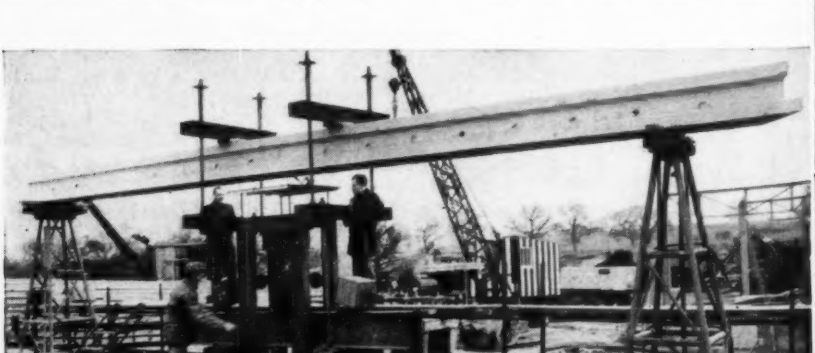


Fig. 11.—Testing a Beam.

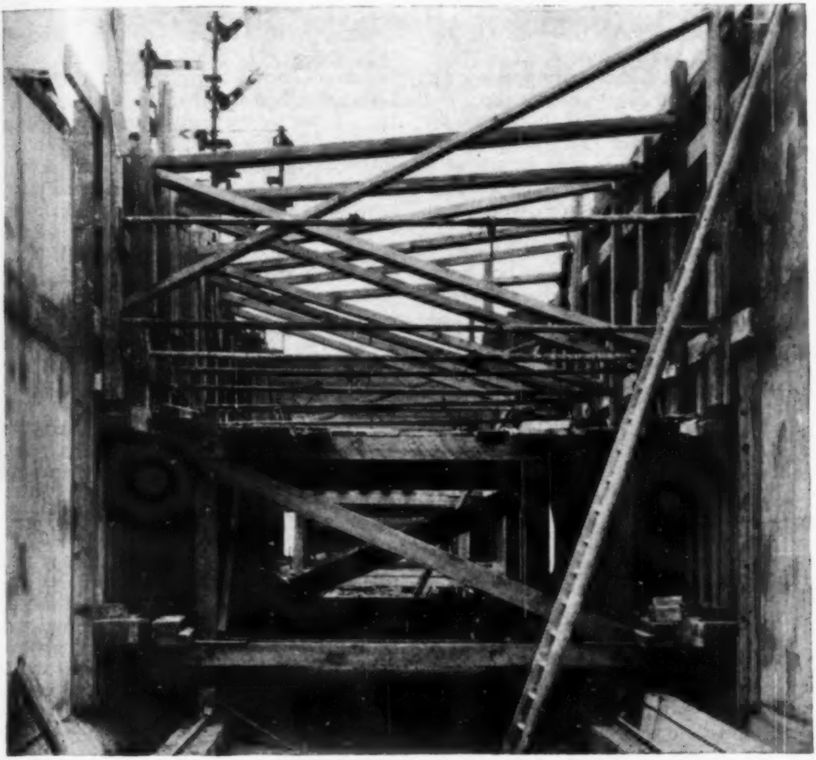


Fig. 12.—Mobile Shuttering for Under-pass.

gravel, and the sides of the excavation are lined with 9-in. brick walls to which two coats of asphalt are applied. The asphalt is carried as a continuous layer on a 12-in. concrete sub-base under the floor. A drainage channel provided along one side of the floor leads to a sump, from which water is removed by three automatic pumps.

The concrete in the walls is placed against the asphalt. The splay and the bottom lift of concrete are placed in ordinary shuttering. A shutter (*Fig.* 12), travelling on a short length of track on the floor, is provided for the main part of the walls. The sheeting is of plastic-faced plywood. Turnbuckles are used to draw the shutter slightly away from the face of the concrete in order to enable the equipment to travel forward to the next position. The walls are of uniform thickness throughout and are 12 in., 18 in. or 24 in. thick according to the height. Expansion joints are provided at intervals of 50 ft., and one construction joint is provided midway between the permanent joints.

Concrete Conveyed Pneumatically.

The floors, walls and roof of the new subway under the tracks and platforms at Barking station form a monolithic reinforced concrete structure. Ramps from the platforms to the subway are also of reinforced concrete. Owing to the restricted space below the tracks, which were in use during construction, the concrete was delivered pneumatically, in batches of 7 cu. ft., through a 6-in. pipe from a plant placed on one side of the station. The plant comprises a 10/7

mixer from which concrete is delivered in batches through a large ball-valve in the top of a steel pressure-vessel. The top is closed and locked, and compressed air from an adjacent compressor enters the vessel at a pressure up to 25 lb. per square inch. The pressure forces the concrete along the pipe, which is sunk in a trench at a level sufficiently low to pass under the tracks. The outlet from the vessel is at the lowest point of the system, as the process is not effective if the pipe slopes downwardly. There are two horizontal and three vertical 90-deg. bends in the pipe between the vessel and the discharge end of the pipe. There is a horizontal travel of about 600 ft. for the ramps and upwards of 1000 ft. to the end of the subway remote from the plant.

The concrete has a slump of about 1½ in. and is discharged on to a banker, and any separation of the materials is remedied by shovelling it into the shuttering and compacting it by vibration. Before passing the first batch of concrete through the pipe, a charge of cement and water between two foamed-rubber dollies is blown through to lubricate the pipe. At the end of a concreting period a dolly is passed through to clear the pipes and is followed by a charge of water. About 3 cu. yd. of concrete can be placed in an hour in awkward positions by this process. Depending on the length of the pipe, there may be two or more batches of concrete in the pipe at one time.

The design and construction of the works are under the general direction of Mr. A. K. Terris, M.I.C.E., Chief Civil Engineer of Eastern Region, British Railways. The general contractors are Messrs. W. & C. French, Ltd., and the prestressed beams were made at the works of this firm at North Farm, Loughton. The piles were provided by Holmpress Piles, Ltd.

Book Reviews.

"Reinforced Concrete in Architecture." By A. A. Rafaat. (New York: Reinhold Publishing Corporation. London: Chapman & Hall, Ltd. Price £6.)

In recent years several books have been published tracing the history of reinforced concrete and giving illustrations to show the trend of design of reinforced and prestressed concrete structures. This is the latest book in this class, and inevitably includes many photographs that have been published often before. It is intended for the use of architects, and its principal purpose (as is the case with other books in this class) is to demonstrate the forms that have been used and to stimulate architects to produce further "imaginative" forms in concrete.

"Allgemeines Iterationsverfahren für Verschiebliche Stabwerke." By Reinhold Glatz. [Berlin: Wilhelm Ernst & Son. Price 24 D.M.]

The author develops a general method by successive approximations for calculating bending moments in multiple-story multiple-bay framed structures with inclined members, including the influence of temperature and settlement of supports, based on the principles of elastic deformation. The translation of joints (sway) is allowed for by assuming that some selected members are not subjected to horizontal displacements and by calculating the external horizontal forces required for equilibrium. It is evident from the examples that much calculation is required for frames subjected to sway. For the detailed study necessary before applying the method, a thorough knowledge of the German language is necessary.

Restraint of Slabs by Edge-Beams.

MR. H. KLASMER, of Tel Aviv, writes as follows.

I wish to make the following comments on Mr. L. S. Müller's article on " Restraint of Slabs by Edge-beams " in your journal for July, 1958.

The author assumes that the slabs span in one direction only at right-angles to the edge-beam, and are composed of strips acting in this direction, say, y. The author's equation for the angle ψ, as is given by equation (1), must be regarded as an approximation to the condition of the entire slab and is of limited validity since it disregards the stiffness of the slab in the other direction, say, x, and the torsional moments acting on the slab between the y-strips. The omission of the effect of torsion leads to serious doubts of the value of Mr. Müller's results.

Consider the slab as a ribbed slab without longitudinal distribution ribs. Now consider the action of one or more distribution ribs, which would load the y-ribs with single loads of varying magnitude and direction, namely, downwardly in the strips near the column strips and upwardly in other strips. Additional terms would have to be introduced in the equation for ψ to allow for these loads. Similar reasoning applies to a solid concrete slab, and it might be possible to estimate approximately the influence on ψ of the torsional moments acting between the column strips and the other strips. It is obvious, however, that in the case of a solid slab the torsional moments have the effect that the least moment of restraint will be nearer $-\frac{wl^2}{12}$ than the value given by equations (6) and (12); similar conclusions apply to equations (18) and (30). Equation (37) and the consequent results for the general case would have to be changed also. Mr. Müller's results are presumably sufficiently reliable for ribbed slabs without strong fillers.

Results similar to those obtained by Mr. Müller were published by F. Halbritter in " Beruecksichtigung der Torsionssteifigkeit von Randtraegern in rahmenartigen Tragwerken " (" Beton-und Stahlbetonbau ", No. 10, 1958), where the torsional moments are also neglected.

Mr. Müller cites my preliminary paper, which was published in final form in the " Journal of the Association of Engineers and Architects in Israel " for March, 1953, and considers two-way reinforced slabs fixed by torsionally-elastic edge-beams. A summary in the English language and diagrams for the moments are given.

MR. L. S. MÜLLER replies:

The justification for considering the stiffness of the slab as in the article was questioned soon after publication by Dr. H. G. Allen. It cannot be denied that a solid concrete slab, even if it is reinforced in one direction only, has some stiffness in the direction of the edge-beam. It is seen from *Fig.* 1 of my article that I visualised a very long narrow slab, uninterrupted by beams at right-angles to the edge-beam. Regarding the behaviour of such long and narrow anisotropic plates, the work of Mr. M. T. Huber gives perhaps more information, but I know no more of his work than is given by Timoshenko in " Shells and Plates " (Chapter V, para. 37). Professor Timoshenko does not give any information on the relative magnitudes of the four elastic constants E_x', E_y', E'', and G when he derives the deflection of an infinitely long plate; therefore the magnitudes of D_x and D_y used in the solution are unknown.

It seems almost certain to me that in a freely-supported concrete slab reinforced in one direction only and very long in the other direction the bending-moment factor is less than 0·0375; that is the moment in the longer direction is certainly less than one-third of the moment in the shorter direction. How much less is questionable. Is the longitudinal stiffness of the slab negligible for the purpose of such an approximate theory? This question can be answered only by tests; all theoretical investigation should be decided by full-scale tests.

Losses of Prestressing Force.

IN Dr.-Ing. Jerzy Zielinski's article on " Losses of Prestressing Force " in the May number of this journal, the formula for S_k on page 169 should read

$$S_k = S_m(1 - l\lambda).$$

Water Meters for Concrete Mixers.

Two types of flow-meters are now available for measuring the amount of water added to concrete, with an accuracy which it is claimed can be within 2 per cent. Each meter is installed in the pipe supplying water.

The manually-operated meter has two indicators; one is set by hand to indicate the amount of water required, the other indicates automatically the amount delivered. When the two indicators coincide, the valve is closed by hand, the flow-indicator is re-set to zero, and the cycle may be repeated. The single indicator of the semi-automatic meter (*Fig.* 1) is set to indicate the quantity of water required. A button is pressed and, when this quantity has been delivered, the valve is closed electrically. The indicator is re-set and the cycle can be repeated.

The meters can be supplied to indicate additionally the total quantity of water used. Between 550 and 4400 gallons per hour may be passed, depending on the diameter of the pipe. The meters are supplied by Messrs. Baumgarten & Angles, of London, who are the agents for Hydrometer Akt. Ges., of Ansbach.

Fig. 1.

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